(1)

Math 201 — Fall 2004–05
Calculus and Analytic Geometry III, sections 5–8
Final exam, January 29 — Duration: 2.5 hours

YOUR NAME:

YOUR AUB ID#:

PLEASE CIRCLE YOUR SECTION:

Section 5	Section 6	Section 7	Section 8
Recitation M 1	Recitation Tu 12:30	Recitation Tu 2	Recitation Tu 3:30
Professor Makdisi	Mr. Khatchadourian	Mr. Khatchadourian	Mr. Khatchadourian

READ THESE INSTRUCTIONS!!

1. PART I of the exam is quick answers. NO JUSTIFICATION REQUIRED, NO PARTIAL CREDIT.
2. PART II of the exam is short answers. No justification required, BUT PARTIAL CREDIT IS AVAILABLE.
3. PART III of the exam is full problems. FULL JUSTIFICATION AND COMPLETE SOLUTIONS ARE REQUIRED, and partial credit is available.
4. Don't forget to enter your NAME, AUB ID number, and SECTION!

GOOD LUCK!

GRADES:

1 (12 pts)	2 (6 pts)	3 (14 pts)	4 (12 pts)	5 (12 pts)	6 (16 pts)
7 (12 pts)	8 (12 pts)	9 (12 pts)	10 (12 pts)	11 (12 pts)	12 (12 pts)

TOTAL OUT OF 144:

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PART I. QUICK ANSWERS, NO JUSTIFICATION REQUIRED, NO PARTIAL CREDIT

1 (2 pts/part, total 12 pts). Which of the following series converge and which diverge? Circle your answer.

1a)	$\sum_{n=0}^{\infty} \frac{2^n}{n!}$	Converges	Diverges
1b)	$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$	Converges	Diverges
1c)	$\sum_{n=1}^{\infty} \frac{3^n + n^3}{4^n}$	Converges	Diverges
1d)	$\sum_{n=1}^{\infty} \sqrt[n]{n}$	Converges	Diverges
1e)	$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\,\ln n}$	Converges	Diverges
1f)	$\sum_{n=1}^{\infty} \frac{\ln(1+1/n^{0.6})}{n^{0.6}}$	Converges	Diverges

2 (2 pts/part, total 6 pts). Fill in the blanks below. (Parts a–c are not related to each other.)

2a) Fill in the blank: the sum of the following series is

$$\sum_{n=0}^{\infty} \frac{4^{n+1}}{5^n} = \underline{\hspace{3cm}}.$$

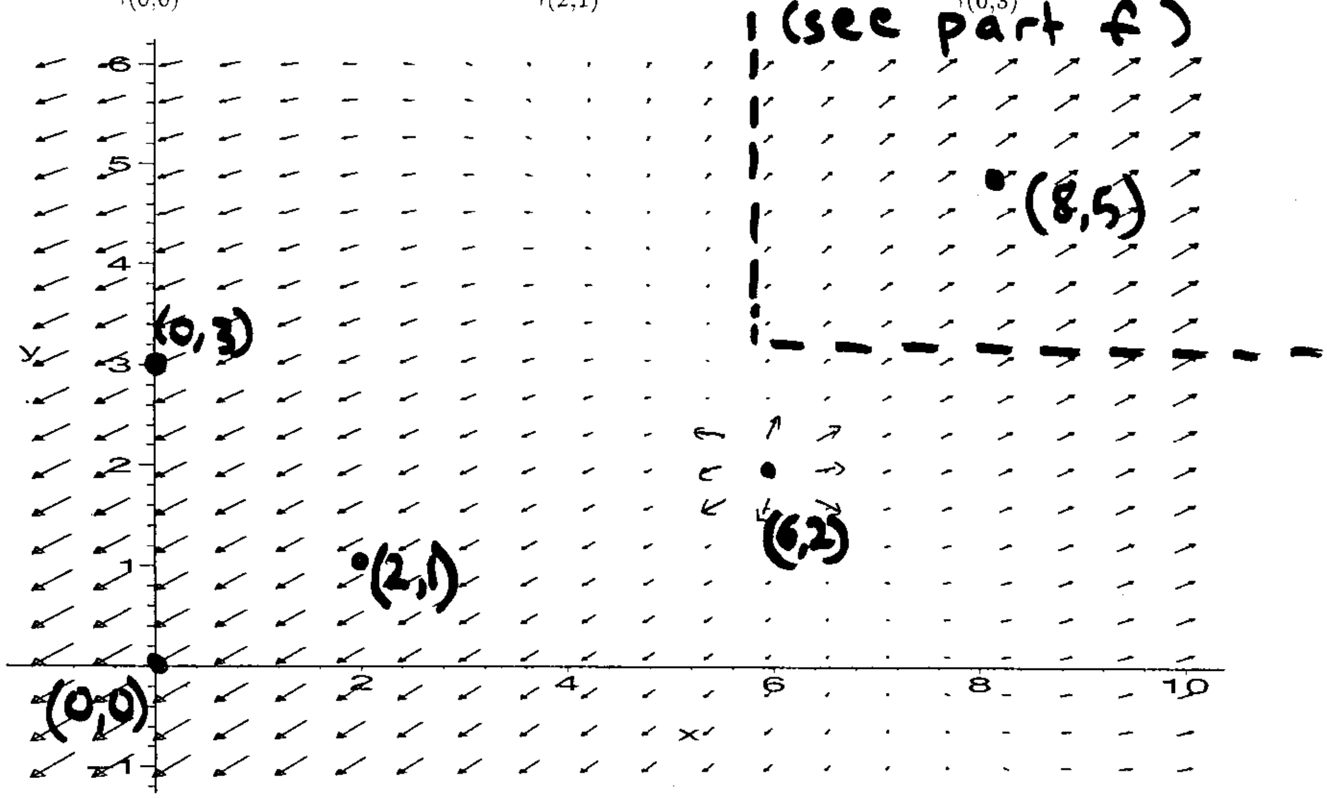
2b) Fill in the blank: $\lim_{x\to 0} \frac{\sin x - x}{x^3} = \underline{\hspace{3cm}}.$

2c) Fill in the blank: the power series $\sum_{n=0}^{\infty} \frac{x^{2n}}{3^n(n^2+1)}$ has radius of convergence

$$R = \underline{\hspace{3cm}}.$$

3 (total 14 pts). The following picture shows the gradient vector field $\vec{\nabla} f$ for a certain function $f(x, y)$. (Note that the gradient vectors are drawn shorter than their true length, to make things easier to visualize.) We also know the following values of the gradient at specific points:

$$\vec{\nabla} f \Big|_{(0,0)} = (-5, -3), \qquad \vec{\nabla} f \Big|_{(2,1)} = (-3, -2), \qquad \vec{\nabla} f \Big|_{(0,3)} = (-4, -2).$$



3a) (2 pts) Which of the following two statements is true? Circle the correct answer.

$f(0, 0) > f(2, 1)$ OR $f(0, 0) < f(2, 1)$

3b) (4 pts) Consider a moving point $P(t) = (t^2 - 2, 2t - 3)$. Fill in the blank for the derivative of $f(P(t))$ at $t = 2$:

$$\frac{d}{dt}\left[f(t^2 - 2, 2t - 3)\right] \Big|_{t=2} = \_\_\_\_\_\_\_\_\_\_\_\_ .$$

3c) (2 pts) The point $(6, 2)$ is a critical point for f. Is it a local maximum, a local minimum, or a saddle point? Circle the correct answer.

Local maximum OR Local minimum OR Saddle point.

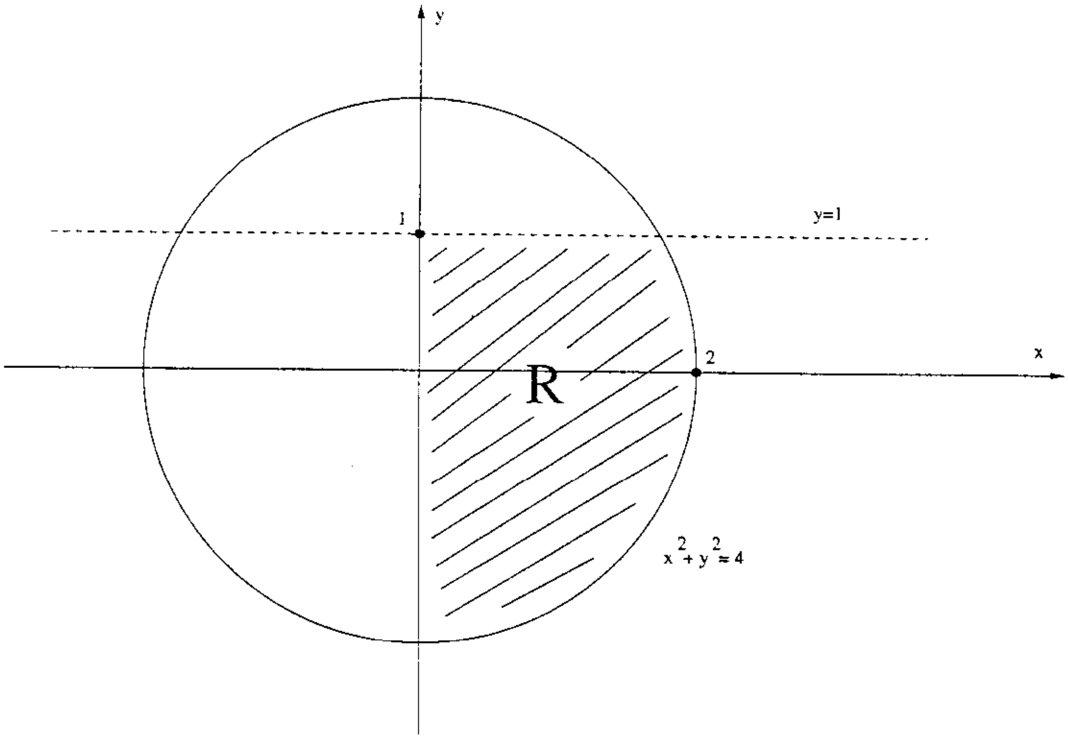
3d)&3e) (4 pts) Use an approximation to circle the expected answer in the two questions below:

3d) The LARGEST number is :	$f(0,3)$	$f(0.01, 3.01)$	$f(0.01, 2.99)$
3e) The SMALLEST number is :	$f(0,3)$	$f(0.01, 3.01)$	$f(0.01, 2.99)$

3f) (2 pts) Draw a **rough sketch** of the level curve passing through the point $(8, 5)$. Please draw **ONLY** the part of the level curve that is inside the box on the top right corner, and don't clutter up the rest of the picture!

PART II. SHORT ANSWERS, no justification needed, PARTIAL CREDIT AVAILABLE.

4 (4 pts each part, total 12 pts). Consider the shaded region R below. Fill in the blanks for the following integrals in rectangular and polar coordinates:



4a) $$\iint_R 1\, dA = \int_{y=\_\_\_\_\_}^{\_\_\_\_\_} \int_{x=\_\_\_\_\_}^{\_\_\_\_\_} dx\, dy.$$

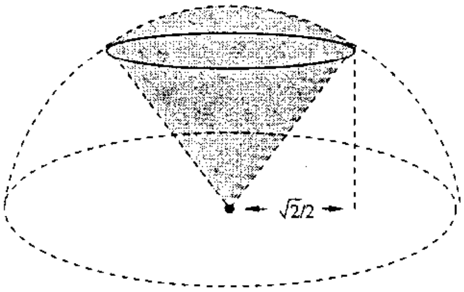
4b) $$\iint_R 1\, dA = \int_{x=\_\_\_\_\_}^{\_\_\_\_\_} \int_{y=\_\_\_\_\_}^{\_\_\_\_\_} dy\, dx$$

$$+ \int_{x=\_\_\_\_\_}^{\_\_\_\_\_} \int_{y=\_\_\_\_\_}^{\_\_\_\_\_} dy\, dx.$$

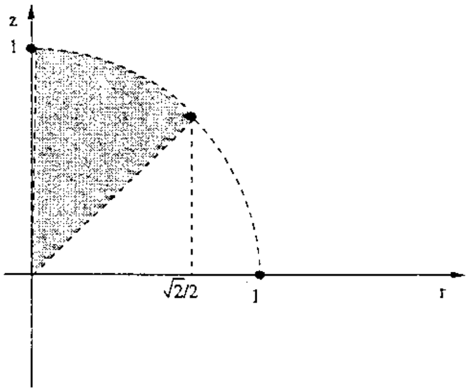
4c) $$\iint_R 1\, dA = \int_{\theta=\_\_\_\_\_}^{\_\_\_\_\_} \int_{r=\_\_\_\_\_}^{\_\_\_\_\_} \_\_\_\_\_\ dr\, d\theta$$

$$+ \int_{\theta=\_\_\_\_\_}^{\_\_\_\_\_} \int_{r=\_\_\_\_\_}^{\_\_\_\_\_} [same]\, dr\, d\theta.$$

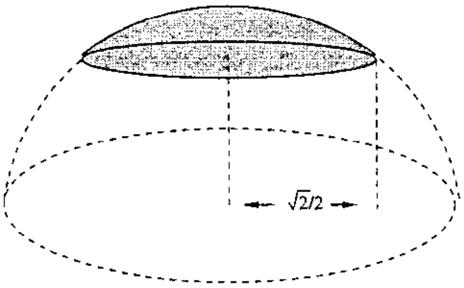
5 (4 pts each part, total 12 pts). In each of the pictures below, we give a 3-dimensional picture and a cross-section of a region D (which is always part of the half-ball $x^2+y^2+z^2 \leq 1, z \geq 0$). In each case, fill in the blanks for integration in spherical coordinates:



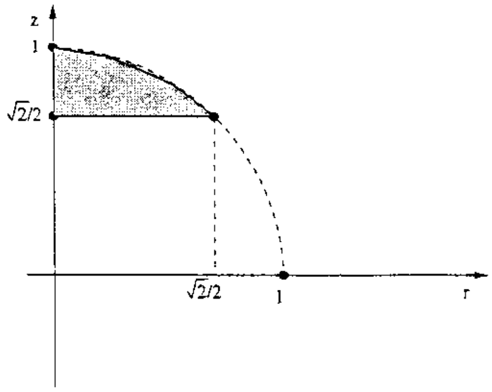
Ice–cream cone



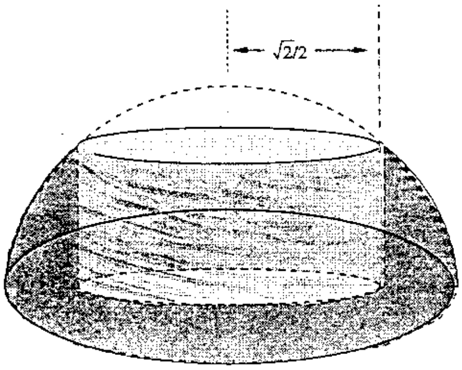
5a) $\displaystyle\iiint_D 1\,dV = \int_{\theta=\_\_\_\_}^{\_\_\_\_} \int_{\varphi=\_\_\_\_}^{\_\_\_\_} \int_{\rho=\_\_\_\_}^{\_\_\_\_} \_\_\_\_\_\_ \, d\rho\, d\varphi\, d\theta.$



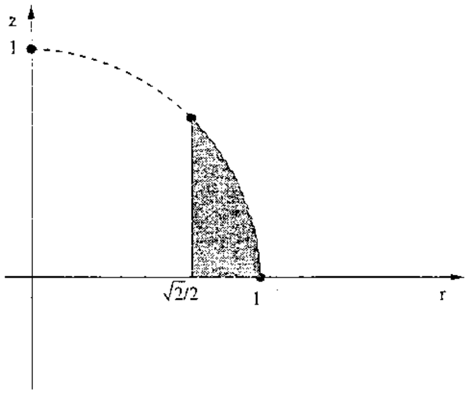
Top cut off by flat plane



5b) $\displaystyle\iiint_D 1\,dV = \int_{\theta=\_\_\_\_}^{\_\_\_\_} \int_{\varphi=\_\_\_\_}^{\_\_\_\_} \int_{\rho=\_\_\_\_}^{\_\_\_\_} [same]\, d\rho\, d\varphi\, d\theta.$



Hollowed–out half–ball
(cylinder of radius $\sqrt{2}/2$ removed)



5c) $\displaystyle\iiint_D 1\,dV = \int_{\theta=\_\_\_\_}^{\_\_\_\_} \int_{\varphi=\_\_\_\_}^{\_\_\_\_} \int_{\rho=\_\_\_\_}^{\_\_\_\_} [same]\, d\rho\, d\varphi\, d\theta.$

6 (4 pts each part, total 16 pts). The parts are not related

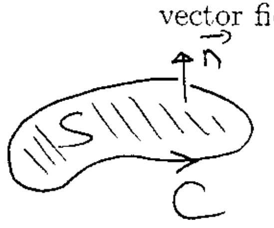
6a) Find the tangent plane to the surface $x^2y + 2yz = 5$ at the point $P(1, 1, 2)$.

6b) Find a potential function for the vector field $\vec{F} = (y, x + z, y + 1)$.

6c) We make a change of variables $x = uw$, $y = u + uw$ to evaluate an integral from a region R in the xy-plane to a region R' in the uw-plane. Fill in the correct value in the blank:

$$\iint_{(x,y)\in R} 2x\, dxdy = \iint_{(u,w)\in R'} \underline{\hspace{6cm}}\, dudw.$$

6d) Given the vector field $\vec{F} = (x^2, xy, yz)$ and a surface S with oriented boundary $C = \partial S$ as shown. Then Stokes' theorem says that $\int_C \vec{F} \cdot d\vec{r} = \iint_S \vec{G} \cdot \vec{n}\, d\sigma$ for a suitable vector field $\vec{G}$. Fill in the blank:



$$\vec{G} = (\underline{\hspace{3cm}}, \underline{\hspace{3cm}}, \underline{\hspace{3cm}}).$$

PART III. FULL SOLUTIONS REQUIRED, PARTIAL CREDIT AVAILABLE.

7 (6 pts each part, total 12 pts).

a) Using power series, express the integral $L = \int_{x=0}^{0.1} e^{-x^3}\, dx$ as a certain alternating series. For full credit, the answer should be written using Σ notation. You can get nearly full credit for just writing out the first four (nonzero) terms in the series.

b) Find (with justification, of course) a specific partial sum s_n for which the error satisfies $|s_n - L| < 10^{-11}$.

Note: in parts (a) and (b), you may use without proof the fact that your series satisfies the conditions of the alternating series estimation theorem.

8 (6 pts each part, total 12 pts).

a) Use Taylor's theorem to find a specific constant A such that for all x with $|x| \leq 1/2$, we have

$$\left|(1+x)^{3/2} - 1 - 3x/2\right| \leq Ax^2. \qquad (*)$$

(You do not have to simplify your expression for A.)

b) Use the inequality $(*)$ to show that $\lim_{(x,y)\to(0,0)} f(x,y) = 0$, where $f(x,y)$ is defined by

$$f(x,y) = \frac{(1+x+y)^{3/2} - 1 - 3(x+y)/2}{\sqrt{x^2+y^2}}.$$

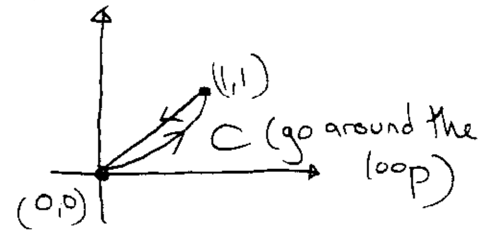
Note: you do not need to know the exact value of A to solve this part. Even if you do know A, you will have a neater solution if you just write the symbol "A" instead of its specific value each time.

9 (total 12 pts). Define the function $f(x, y) = x^2 + 2y^2 - y^3/3$.

a) (4 pts) Find and classify the critical points of f.

b) (8 pts) We **constrain** (x, y) to lie in the disk $x^2 + y^2 \leq 1$. At what points of the disk does f attain its maximum and its minimum values?

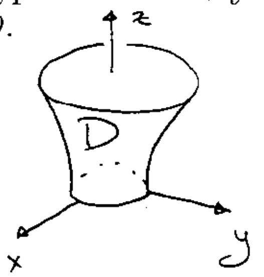
10 (6 pts each part, total 12 pts). Let C be the closed curve in the plane starting from $(0,0)$, which first goes to $(1,1)$ along the parabola $y = x^2$, and then returns to $(0,0)$ along the line $y = x$. Compute $\int_C y\,dx + e^y\,dy$ **twice**: (a) directly, (b) using Green's theorem.



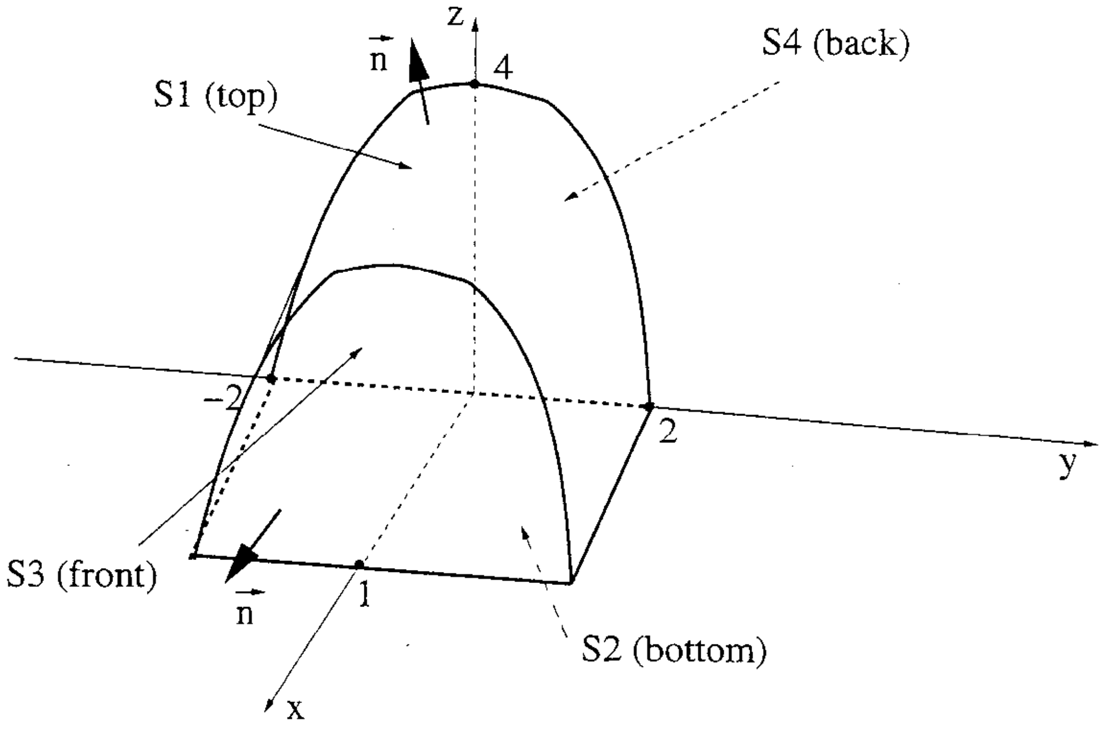
Solution for (a):

Solution for (b):

11 (12 pts). Let D be the solid region in space, shaped like a modern table, which is bounded below by the plane $z = 0$, above by the plane $z = 1$, and on the sides by the hyperboloid $x^2 + y^2 - z^2 = 1$. The density of D is $\delta(x, y, z) = 2z$. Find the total mass of D.



12 (**3 parts**, total 12 pts). Let D be the solid region in space which is bounded above by the surface $S_1 : z = 4 - y^2$, below by the plane $S_2 : z = 0$, and on the front and back by the planes $S_3 : x = 1$ and $S_4 : x = 0$, respectively. Define the vector field $\vec{F}\Big|_{(x,y,z)} = (x^2, 0, z)$. (This is the same as saying that $\vec{F} = x^2\mathbf{i} + z\mathbf{k}$.) The normal vector on the surfaces $S_1, \ldots, S_4$ is oriented outwards from D.



a) (6 pts) Compute the flux integral $\iint_{S_1} \vec{F} \cdot \vec{n}\, d\sigma$.

CONTINUED ON NEXT PAGE!! Parts (b) and (c) are overleaf.

Continuation of 12.

b) (2 pts) Set up but **do not evaluate** the flux integral $\iint_{S_3} \vec{F} \cdot \vec{n}\, d\sigma$.

c) (4 pts) Set up **but do not evaluate** a triple integral over the solid D which is equal by the Divergence Theorem to $\iint_{S_1} \vec{F} \cdot \vec{n}\, d\sigma + \iint_{S_2} \vec{F} \cdot \vec{n}\, d\sigma + \iint_{S_3} \vec{F} \cdot \vec{n}\, d\sigma + \iint_{S_4} \vec{F} \cdot \vec{n}\, d\sigma$.